# Calculation of 3D Ground Settlement of Double-line Tunnel with Variable Cross-section 

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#### Abstract

Based on the PECK formula, the two-dimensional settlement calculation of the stratum under the tunnel excavation is extended to three-dimension, and the three-dimensional solution of the ground settlement under the influence of the superposition of the tunnel excavation of each section under the double-line tunnel variable section excavation is analyzed and analyzed. It shows that under the action of superimposed disturbance of double-line construction, the settlement of the ground surface mainly shows displacement to the tunnel excavation, and the settlement of the ground appears settlement valley directly below the excavation axis of the double-line tunnel; the building structure has a significant effect on the settlement of the surface and the effective rigidity The larger, the wider and shallower the settlement trough. At the transition of excavation, the surface settlement is prone to "points".


Keywords: PECK formula; double-line tunnel; variable-section tunnel; three-dimensional settlement degradation

## I. INTRODUCTION

In the process of tunnel construction, the formation loss caused by the excavated rock mass, the release of formation stress, the mechanical effect, structural effect and drainage consolidation during construction are the main factors causing formation settlement. However, with the increase of demand, the working conditions of tunnel projects have become more and more complex. Multi section and multi construction method transformation construction frequently occurs in tunnel construction projects. How to measure the stratum settlement under the combination of multi working conditions has become the main problem of complex tunnel construction.

For the above factors, many scholars have done a lot of research on each factor. For natural ground, Peck ${ }^{[1]}$ put forward the famous empirical
formula of settlement curve through sorting and studying a large number of measured data. It is considered that the stratum settlement trough curve caused by tunnel excavation follows the normal distribution on the cross section; Clough and Schimidt ${ }^{[2]}$ of the United Kingdom, through the study of soft clay tunnel, in their works on soft clay tunnel engineering, gave the value calculation method of surface settlement trough width parameter for saturated soft clay soil layer; Attewell et al. ${ }^{[3]}$ comprehensively referred to the research results of Peck ${ }^{[1]}$ and clough and Schimidt ${ }^{[2],}$ and gave the empirical formula applicable to different soil layers when the surface settlement is a normal distribution curve; Osman et al. ${ }^{[4]}$ made further research on surface settlement and put forward the value formula of surface settlement trough width parameters at different depths; The Polish scholar J. litviniszyn put forward the random medium theory, holding that the rock and soil mass is a random medium, and the settlement and displacement of the stratum after excavation is a random process due to the generation of free surface, Chinese scholars (Yang Junsheng, Liu Baochen, $1998{ }^{[5]}$; Yang Junsheng, Liu Baochen, 2002 ${ }^{[6]}$; Shi Chenghua, Peng Limin, Liu Baochen, $2003{ }^{[7])}$ have further improved and developed this theory and widely applied it to the calculation of stratum displacement and settlement caused by underground mining, urban subway tunnel excavation and other underground projects; Attewell and Woodman ${ }^{[3]}$ expanded the stratum settlement curve under tunnel excavation from describing only the vertical tunnel excavation direction to describing the three-dimensional curve along the tunnel excavation direction in 1982, and deduced the three-dimensional calculation formula of stratum settlement trough under tunnel excavation; Most of the above studies are about the surface transverse settlement caused by tunnel excavation, and the three-dimensional
expression of settlement tank proposed by Attewell and Woodman［3］is only applicable to the tunnel with single section．For some special sections，the tunnel sections show diversified combination modes，and the existing theories are difficult to be simply applied to the more complex and changeable engineering environment．Therefore，it is necessary to put forward the three－dimensional subsidence degradation of stratum with wider application range，so as to provide reference for tunnel construction with similar complex and variable cross－section．

Analytical solution of surface settlement caused by variable section tunnel If the volume change caused by soil compression in the process of surface settlement is ignored，and only the formation settlement is caused by the soil loss in the process of opening excavation，the displacement change at any point of the formation can be regarded as the cumulative sum of displacement caused by the displacement sources at each point in the process of tunnel excavation，According to the three－dimensional calculation formula of stratum settlement tank under tunnel excavation proposed by Attewell and Woodman［3］in 1982，the surface settlement caused by any point displacement source is：
$s=\frac{A V_{1}}{\sqrt{2 \pi} K z} \exp \left[\frac{-\left(\mathrm{x}^{2}+\mathrm{y}^{2}\right)}{2(\mathrm{Kz})^{2}}\right]$
Where ：s－Surface settlement caused by point displacement source ；

A－Tunnel excavation section ；
$\mathrm{V}_{\mathrm{l}}$－Formation loss rate ；
K－Sedimentation tank width coefficient ；
z－Buried depth of tunnel axis。
The above formula（1）gives the surface settlement value of the tunnel face at a special location， but in the actual project，the tunnel face is a dynamic change process of moving forward continuously．If the position of the tunnel face is $x=0$ and the positive direction of X axis along the tunnel excavation direction，establish the Cartesian space rectangular coordinate system $\mathrm{z}-\mathrm{x} 0 \mathrm{y}$ ，taking the starting point of tunnel excavation as，The Cartesian space rectangular coordinate system Z －is established along the positive axis of the tunnel excavation direction．According to the relative position relationship between the two coordinate systems，in the space rectangular coordinate system Z －，equation（1）becomes：
$\mathrm{s}=\frac{\mathrm{AV}}{\sqrt{2}} \frac{\sqrt{2 \pi \mathrm{Kz}}}{} \exp \left[\frac{-\left(\left(\mathrm{x}-\mathrm{x}^{\prime}\right)^{2}+\left(\mathrm{y}^{\prime}\right)^{2}\right)}{2(\mathrm{Kz})^{2}}\right]$
The above equation（2）is obtained by integrating from the starting point of tunnel excavation to the end point of tunnel excavation along the tunnel excavation direction ：
$\mathrm{s}=$
$\int_{\mathrm{X}_{\mathrm{i}}}^{\mathrm{X}_{\mathrm{f}}} \frac{\mathrm{AV}_{1}}{\sqrt{2 \pi \mathrm{Kz}}} \exp \left[\frac{-\left(\left(\mathrm{x}-\mathrm{x}^{\prime}\right)^{2}+\left(\mathrm{y}^{\prime}\right)^{2}\right)}{2(\mathrm{Kz})^{2}}\right]=$
$\frac{\mathrm{AV}_{1}}{\sqrt{2 \pi \mathrm{Kz}}} \exp \left[\frac{-\left(\mathrm{y}^{\prime}\right)^{2}}{2(\mathrm{Kz})^{2}}\right]\left\{G\left(\frac{\mathrm{x}-\mathrm{x}_{\mathrm{i}}}{\mathrm{Kz}}\right)-\mathrm{G}\left(\frac{\mathrm{x}-\mathrm{x}_{\mathrm{f}}}{\mathrm{Kz}}\right)\right\}$
where $x_{i}$－In the space rectangular coordinate system $z-x 0 y$ ，the starting point position of the tunnel ；
$\mathrm{x}_{\mathrm{f}}$－In the space rectangular coordinate system $\mathrm{z}-\mathrm{x} 0 \mathrm{y}$ ， the position of the end point of the tunnel，and the subscript indicates the position of the starting point of the tunnel；Subscript indicates the position of tunnel excavation surface during calculation；
G－The definition of Gprobability function is shown in the following formula ：
$G(\alpha)=\frac{1}{\sqrt{2 \pi}} \int_{-\infty}^{\alpha} \exp \left[\frac{-\beta^{2}}{2}\right] d \beta$（4）
Especially，$G(0)=\frac{1}{2}, G(\infty)=0$ 。
The above formula（3）is the displacement expression caused by the stratum of a constant section tunnel from the excavation starting face to the end face． Since the coordinates in the coordinate system Z－are consistent with the Y coordinates in the coordinate system z－x0y，the formula（3）can be rewritten as：
$s=\frac{\mathrm{AV}_{1}}{\sqrt{2 \pi \mathrm{Kz}}} \exp \left[\frac{-\mathrm{y}^{2}}{2(\mathrm{Kz})^{2}}\right]\left\{\mathrm{G}\left(\frac{\mathrm{x}-\mathrm{x}_{\mathrm{i}}}{\mathrm{Kz}}\right)-\mathrm{G}\left(\frac{\mathrm{x}-\mathrm{x}_{\mathrm{f}}}{\mathrm{Kz}}\right)\right\}$
（5）
Equation（5）is the expression of three－dimensional settlement and displacement of stratum caused by constant section excavation under the unified coordinate system．On the basis of equation（5），for variable section tunnel in special section，the three－dimensional settlement of stratum caused by tunnel excavation can be expressed by the following formula：$s=\sum_{k} \frac{A_{k} V_{l}}{\sqrt{2 \pi K_{k} Z}} \exp \left[\frac{-y^{2}}{2\left(K_{k} Z\right)^{2}}\right]\left\{G\left(\frac{\left(x-l_{k}\right)-x_{i}}{K_{k} Z}\right)-\right.$ Gx－lk－xfKkz

Where ： k －Tunnel variable section code ；
$\left(\mathrm{V}_{1}\right)_{\mathrm{k}}$－Tunnel stratum loss of section K ；
$\mathrm{K}_{\mathrm{k}}$－Width coefficient of settlement trough caused by section K tunnel ；
$\mathrm{l}_{\mathrm{k}}$－Distance from the starting point of section K tunnel to the origin $(\mathrm{x}=0)$ 。

Equation（6）is the expression of three－dimensional settlement deformation of the stratum caused by the combined construction of variable section tunnel under the single track tunnel．It should be noted that the above equation（6）only considers the influence of simple superposition disturbance under different section combinations，and does not consider the influence of secondary disturbance caused by the rear excavation face to the front excavation section during the combined construction of different sections．In fact，for the

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double track parallel tunnel, if only the simple superposition effect between the two tunnels is considered and the influence of the trailing line on the secondary disturbance caused by the leading line is ignored, the expression of three-dimensional settlement deformation caused by the excavation of double track variable section tunnel can be obtained on the basis of equation (6) :
$\mathrm{s}=\sum_{\mathrm{kL}} \frac{\mathrm{A}_{\mathrm{kL}} \mathrm{V}_{\mathrm{l}}}{\sqrt{2 \pi \mathrm{~K}_{\mathrm{kL}} \mathrm{Z}}} \exp \left[\frac{-\mathrm{y}^{2}}{2\left(\mathrm{~K}_{\mathrm{kL}}\right)^{2}}\right]\left\{\mathrm{G}\left(\frac{\left(\mathrm{x}-\mathrm{l}_{\mathrm{kL}}\right)-\mathrm{x}_{\mathrm{iL}}}{\mathrm{K}_{\mathrm{kL}} \mathrm{Z}}\right)-\right.$
Gx-lkL-xfLKkLz+kRAkRVl2 $\pi$ KkRzexp-y22KkRz
2Gx-lkR-xiRKkRz-Gx-lkR-xfRKkRz
(7)

Where : Subscript indicates the left line of the tunnel during calculation; Subscript indicates the right line of the tunnel.

The above formula (7) is the expression of three-dimensional settlement displacement of stratum under the construction of double track parallel variable cross-section tunnel.

Relevant research shows that the settlement displacement of the stratum has a great relationship with the buried depth of the tunnel. The greater the buried depth of the tunnel, the wider and shallower the settlement trough, the smaller the buried depth of the tunnel, and the narrower and deeper the settlement trough. Based on the above research, if the principle of equivalent stiffness is introduced, the influence of building structure stiffness can be considered. For the building structure with thickness T and structural stiffness m , it can be equivalent to the influence of
overlying soil layer with thickness $\mathrm{h}_{\mathrm{s}}$ and stiffness $\mathrm{M}_{\mathrm{s}}$, namely:

$$
\begin{equation*}
\mathrm{t} \cdot \mathrm{M}=\mathrm{h}_{\mathrm{s}} \cdot \mathrm{M}_{\mathrm{s}} \tag{8}
\end{equation*}
$$

At this time, the embedded depth of tunnel excavation axis is equivalent to :
$z_{s}=z+\frac{h_{s} \cdot M_{s}}{M}$
$\frac{\mathrm{z}_{\mathrm{s}}}{\mathrm{z}}=1+\frac{\mathrm{h}_{\mathrm{s}} \cdot \mathrm{M}_{\mathrm{s}}}{\mathrm{z} \cdot \mathrm{M}}$
Define the correction coefficient of settlement tank width after considering the stiffness of building structure. In addition:
$\eta=\frac{\mathrm{h}_{\mathrm{s}} \cdot \mathrm{M}_{\mathrm{s}}}{\mathrm{z} \cdot \mathrm{M}}$
The expression of three-dimensional settlement deformation caused by double track variable section tunnel excavation under the action of building structure stiffness is :

$$
\begin{align*}
& s=\sum_{\mathrm{kL}} \begin{array}{l}
\frac{\mathrm{A}_{\mathrm{kL}} V_{\mathrm{l}}}{\sqrt{2 \pi\left(1+\eta_{\mathrm{KL}}\right) \mathrm{K}_{\mathrm{kL}} \mathrm{z}}} \exp \left[\frac{-\mathrm{y}^{2}}{2\left(\left(1+\eta_{\mathrm{KL}}\right) \mathrm{K}_{\mathrm{kL}} \mathrm{z}\right)^{2}}\right] \\
\left\{\mathrm{G}\left(\frac{\left(\mathrm{x}-\mathrm{l}_{\mathrm{kL}}\right)-\mathrm{x}_{\mathrm{iL}}}{\left(1+\eta_{\mathrm{KL}}\right) \mathrm{K}_{\mathrm{kL}} \mathrm{z}}\right)-\mathrm{G}\left(\frac{\left(\mathrm{x}-l_{\mathrm{kL}}\right)-\mathrm{x}_{\mathrm{fL}}}{\left(1+\eta_{\mathrm{KL}}\right) \mathrm{K}_{\mathrm{kL}} \mathrm{z}}\right)\right\}
\end{array}+ \\
& \sum_{\mathrm{kR}} \begin{array}{l}
\frac{A_{\mathrm{kR}} \mathrm{~V}_{1}}{\sqrt{2 \pi\left(1+\eta_{\mathrm{KR}}\right) \mathrm{K}_{\mathrm{kR}} \mathrm{Z}}} \exp \left[\frac{-\mathrm{y}^{2}}{2\left(\left(1+\eta_{\mathrm{KR}}\right) \mathrm{K}_{\mathrm{kR}} \mathrm{z}\right)^{2}}\right] \\
\left\{\mathrm{G}\left(\frac{\left(\mathrm{x}-\mathrm{l}_{\mathrm{kR}}\right)-\mathrm{x}_{\mathrm{iR}}}{\left(1+\eta_{\mathrm{KR}}\right) \mathrm{K}_{\mathrm{kR}} \mathrm{z}}\right)-\mathrm{G}\left(\frac{\left(\mathrm{x}-l_{\mathrm{kR}}\right)-\mathrm{x}_{\mathrm{R}}}{\left(1+\eta_{\mathrm{KR}}\right) \mathrm{K}_{\mathrm{kR}} \mathrm{z}}\right)\right\}
\end{array} \tag{12}
\end{align*}
$$

## II. EXAMPLE ANALYSIS

For the concealed excavation section of the left and right lines of a project, the construction of the left line shall be carried out first, and then the construction of the right line. The length and excavation span of each construction section in the section are shown in Table 1.

Table 1 description of section

| position | Starting mileage | Section type | Section size |  |  | Excavation method |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | length/m | span/m | height <br> /m |  |
| Left line | ZDK33+457.095~ZDK33+464.705 | A | 7.61 | 8.24 | 8.44 | Step method |
|  | ZDK33+510.462~ZDK33+574.765 | A | 64.303 | 8.24 | 8.44 | Step method |
|  | ZDK33+442.495~ZDK33+457.095 | B | 14.6 | 10.84 | 10.89 | Step method |
|  | ZDK33+464.705~ZDK33+492.705 | C | 28 | 12.118 | 9.41 | CD method |
|  | ZDK33+492.705~ZDK33+510.462 | D | 17.757 | 16.147 | 10.679 | CRD method |
| Right line | YDK33+558.765~YDK33+574.765 | A | 16 | 8.24 | 8.44 | Step method |
|  | YDK33+532.079~YDK33+558.765 | C | 26.686 | 12.118 | 9.41 | CDmethod |
|  | YDK33+517.279~YDK33+532.079 | D | 19.8 | 16.147 | 10.679 | CRD method |
| Transition section of left and | $\begin{aligned} & \text { YDK33+510.462~ } \\ & \text { YDK33+517.279 } \end{aligned}$ | E | 6.817 | 6.4 | 6.79 | Step method |

According to the results summarized and discussed in document [8], $\mathrm{V}_{1}=2 \%, \mathrm{~K}=0.5$ is taken. According to document [9], the correction coefficient of settlement tank width is $\mathrm{t} \eta=1.31405$. After
calculation, the three-dimensional surface settlement diagrams without considering and considering the stiffness of building structure are obtained, as shown in Figure 1 and 2 below.


Fig. 1 three dimensional displacement diagram of surface settlement under tunnel excavation without considering the structural stiffness of buildings

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Fig. 2 three dimensional displacement diagram of surface settlement under tunnel excavation considering the structural stiffness of buildings

The surface settlement is in the X direction, that is, perpendicular to the tunnel excavation direction, and there are settlement peaks at $\mathrm{x}=0$ and $\mathrm{x}=17.2 \mathrm{~m}$, which are just above the tunnel axis of the left line and the right line, which is consistent with our cognition; Under the superimposed disturbance of double track construction, the surface mainly tends to move to the tunnel excavation section. In the process of tunnel excavation, if there is large section excavation, there will be "sharp points" in the stratum displacement. Special attention should be paid to strengthening the surface settlement monitoring at the control points.

Under the constraint of the building structure, it is obvious that under the constraint of the building structure stiffness, the surface settlement decreases obviously. Along the tunnel excavation direction, the double peaks of the settlement of the double track tunnel are not obvious, mainly because the left and right lines are close to each other and have a great influence on each other. Under the constraint of the structural stiffness, the settlement displacement tends to accumulate in one place, which is perpendicular to the tunnel excavation direction, There are still two peaks in vertical settlement, which shows that the influence of different excavation
surfaces on stratum displacement is very significant.

## III. CONCLUSION

On the basis of peck formula and considering the superposition between tunnels under close excavation, this paper obtains the three-dimensional calculation expression of surface settlement under double track and multi section close excavation, and draws the following conclusions:

1) For the double track tunnel, two settlement troughs will appear in the surface settlement, which are located directly below the tunnel excavation axis. Under the superposition disturbance of double track construction, the surface settlement mainly shows the displacement to the tunnel excavation;
2) The stiffness of the building structure has a significant "constraint" effect on the surface settlement, and the building structure has a significant impact on the surface settlement. The greater the effective stiffness is, the wider and shallower the settlement trough is. When the spacing of the double track tunnels is small, due to the stiffness of the building structure, the settlement peaks of the double track tunnels are not obvious along the tunnel excavation direction, and the settlement displacement tends to accumulate in one
place;
3) The surface settlement at the excavation transition of different sections will change obviously, and the surface settlement displacement is prone to "sharp points" here. Special attention should be paid to strengthening the surface settlement monitoring at the control points.

## REFERENCES

[1]. PECK R B. Deep excavation and tunneling in soft ground [ C ] . Proceeding of 7th International Conference on Soil Mechanics and Foundation Engineering. Mexico City: [s. n. ], 1969: 225.
[2]. Clough G W, Schmidt B. Design and Performance of Excavations and Tunnels in Soft Clay. In Soft Clay Engineering, Elsevier. 1981,569-634
[3]. Attewell P B, Woodman J P. Predicting the dynamics of ground settlement and its derivatives by tunneling in soil. Ground Engineering, 1982,15(8):13-22
[4]. Osman A S, Mair R J, Bolton M D. On the kinematics of 2D tunnel collapse in undrained clay. Geotechnique, 2006,56(9):585-595
[5]. Yang Junsheng, Liu Baochen Surface movement and deformation caused by extrusion shield tunnel construction, geotechnical mechanics, Volume 19, issue 3, September 1998, $10 \sim 13$
[6]. Yang Junsheng, Liu Baochen Ground movement and deformation caused by urban tunnel construction Beijing: China Railway Press January 2002
[7]. Shi Chenghua, Peng Limin, Liu Baochen Longitudinal stratum movement and deformation caused by shallow tunnel construction, China Railway Science, Vol. 24, No. 4, August 2003, 87 ~ 91
[8]. Han Xuan Study on practical method for prediction of stratum displacement and building deformation caused by tunnel construction [D] Xi'an University of technology, 2007
[9]. Fu Jinyang Analysis of influence of shallow buried underground excavation subway construction on adjacent buildings in water rich composite stratum [D] Central South University, 2010

